On the Extraction of Scientific Insight from Numerical Simulations of Stratified and Rotating Turbulence
What is Turbulence?

Turbulence is a ubiquitous phenomenon of scientific and engineering significance

- In our bodies (cardiac and respiratory flows)
- Wakes of airplanes and cars
- Winds and currents of the ocean and atmosphere

Turbulent flows have several universal properties

- Turbulent flows appear irregular or 'random'
- Enhanced mixing (milk in tea)
- Strong presence of dissipation

www.wikipedia.org
Turbulence contains a wide range of dynamically important length scales

- Energy containing range
- Dissipation range
- Inertial range
The Multiscale Nature of Turbulence

Turbulence contains a wide range of dynamically important length scales:

- Energy containing range
- Dissipation range
- Inertial range

\[ E_{II}(k_l)/\langle e^2 \rangle^{1/4} \]

Increasing Reynolds Number

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Geophysical Turbulence

Geophysical flows differ fundamentally from many engineering flows

- Large scale density gradients or stratification greatly affect dynamics
- The effects of the Earth's rotation can be non-negligible
- Rotation/stratification leads to coherent vortical structures
- Wider range of dynamically significant length scales (1,000km horizontal and 1km vertical down to 1mm)

Why care?

- The ocean plays a pivotal role in the Earth's climate due to the strong coupling of the atmosphere and ocean
- Accurate parametrization of the smaller scales is needed to predict behavior of large scale currents

Computational methods are ideal to explore these flows

- No analytical solutions exist for turbulent flows
- Prohibitive experimental cost and complexity
Computational Fluid Dynamics

There are three basic approaches used in CFD to predict fluid flow

Reynolds-Averaged Navier-Stokes (RANS)

- Strong dependence on empirical models, largest possible domain sizes

Large Eddy Simulation (LES)

- Some dependence on empirical models, only reasonable for smaller domains

Direct Numerical Simulation (DNS)

- Solves the very accurate unsteady 3D Navier-Stokes equations (below), only small domains possible

\[
\frac{\partial u_i}{\partial t} + \frac{\partial [u_i u_j]}{\partial x_j} + \epsilon_{ijk} \frac{\delta_{j3}}{Ro} u_k = - \frac{\partial p}{\partial x_i} - R_i \delta_{i3} \rho + \frac{1}{Re_0} \frac{\partial^2 u_i}{\partial x_j \partial x_j}
\]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial [\rho u_j]}{\partial x_j} = \frac{1}{Re_0 Pr} \frac{\partial^2 \rho}{\partial x_j \partial x_j}
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\[
\begin{align*}
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\frac{\partial \rho}{\partial t} + \frac{\partial [\rho u_j]}{\partial x_j} &= \frac{1}{Re_0 Pr} \frac{\partial^2 \rho}{\partial x_j \partial x_j}
\end{align*}
\]
Massively Parallel Algorithms

Fast serial algorithms tend to scale poorly when run in parallel on a distributed memory system.

Simulating incompressible fluids requires the solution of elliptical PDEs:
- The flow at every point depends on information from every other point.
- Multigrid methods and/or Fourier based methods are generally used.

Domain decomposition:
- Generally pencil (left) or block decompositions are required for linear scaling.
- Data transposes (left) can be used and still have linear scaling into the 10,000s of processes.

Various domain configurations.
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Computational Details

Simulations performed on Triton Compute Cluster (TCC @ SDSC)
- 2 quad-core Intel Nehalem 2.4 GHz processors, 24 GB memory per node
- 256 cores (32 nodes), ~10k core-hours per case
- Over 900 million computational points used (1536 x 768 x 768)

3D visualization performed on Petascale Data Analysis Facility (PDAF @ SDSC)
- Each snapshot contains 44GB of flow data
- 8 quad-core Shanghai 8380 2.5 GHz processors, 256/512 GB memory per node

Database operations performed on Dash (@ SDSC)
- Two quad-core 2.4 GHz Intel Nehalem processors per node (512 compute cores)
- 7 TB total memory (DRAM+flash)
Submesoscale Dynamics

The submesoscales are some of the least understood scales in the ocean

- Larger scales (>10km) have a far broader base of theory explaining important dynamics and detailed observational data is far more plentiful

- Smaller scales (<10m) behave in manners consistent with many engineering applications with more 'classical' turbulent behavior

- The effects of rotation and stratification are significant in this range unlike smaller scales, but do not dominate dynamics as observed in larger scale flows

The submesoscale range is important

- Submesoscale features are the pathway from large scale currents to small scale turbulent mixing

JPO 38, 44–64. 2008
Observational and numerical data suggest coherent structures play an important role in submesoscale dynamics. An objective definition of a coherent structure is desired for the analysis. The $\lambda_c$ criterion is used to isolate coherent structures from incoherent background flow.

**Model Formulation**

Two horizontally oriented streams subjected to vertical stratification.

The frame of reference can be rotated about the vertical axis.

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The $\lambda_2$ criterion is used to isolate coherent structures from incoherent background flow.

- Defines vortices as regions of the flow field rotating within a plane

\[
S_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right], \quad \Omega_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right],
\]

\[
\lambda_2 = \lambda_2 (S_{ik} S_{kj} + \Omega_{ik} \Omega_{kj})
\]
Coherent Structure Evolution
In addition to allowing visualization of coherent structures, $\lambda_2$ allows quantification of the influence of coherent structures. This ability has provided insight into the role of coherent structures in the stratified horizontal shear layer.

- Vortical structures play important roles in the coupling of scalar and velocity fields, containing concentrations of kinetic and potential energies.

- Energy transport is primarily accomplished by the ambient flow.
Wavelet Analysis

Wavelets have compact support both in physical space and wavenumber space making them a useful tool for analysis of multi-scale physics.

The vorticity field was split into a coherent and incoherent (random) field using wavelet filtering.

- Coiflet wavelets with six vanishing moments are used.

The coherent field contained only 0.346% of wavelet coefficients late in the simulation (3% is typical for homogeneous turbulence).

- Turbulence statistics were very well resolved by the coherent field.

Coherent and incoherent vorticity fields.
Saaz : A Computational Database
Alden King and Scott Baden

Overcome relational DB’s limitations

- Conserve locality in tightly nested, computational loops
- Irregular data

Ease gathering of statistics (queries)

- Domains of interest travel with the queries

Optimizes and parallelizes shared memory queries and eventually distributed memory queries
Saaz Constructs
Alden King and Scott Baden

Domains

- Computational grid ($\mathbb{Z}^n$)

- Computationally-defined structures (Irregular subsets of $\mathbb{Z}^n$)

- Iterators facilitate changes in which direction(s) are inhomogeneous

Multidimensional dense arrays defined on $\mathbb{Z}^n$

Queries over domains

- Statistics within domains

- Conditional: restrict to an arbitrary set of points

- Changing domains is simple due to abstraction of domain characteristics
Other problems currently being explored in the UCSD Computational Fluid Dynamics Laboratory
The Equatorial Undercurrent (EUC) system is dynamically complex

- Understanding of vertical mixing and surface flux is critical in building the global ocean circulation model

Observations show internal waves and intermittent, coherent patches of intensive mixing in a nominally stable region but there is no definitive explanation regarding the physical processes that drives mixing

Our preliminary EUC model using DNS shows that the mixing is caused by coherent horseshoe vortices that penetrate into the stable region.
Oscillating Flow over a Sloping Boundary

Bishakhdutta Gayen

Turbulence and mixing are observed along ridges and continental slopes but their characteristics and relation with respect to the oscillating tide are not understood.

DNS and LES are used to explore the behavior of tidal flow over sloping topography.

Finite difference method with conformal mapping is employed.

Intense turbulence near the bottom is found in the vicinity of specific slope angle (so-called critical angle that has resonant wave response) and at specific phases of the tide.

30 million grid points used.

Energy contour plot.
The Immersed Boundary Method

The IBM method allows the use of existing Navier-Stokes solvers to begin exploring flows around complex geometries with relatively little modification.

Easy to deal with complex geometries that are very difficult to deal with using other solvers (FVM, FD with conformal mapping, etc.)

Works for any arbitrary topography

Avoids generation of complex grids

Flow over a sphere

Internal waves generated by an oscillating cylinder
The Stratified Wake
Matt de Stadler

The stratified wake behind a bluff body is a problem of environmental and engineering interest

- In stratified environments wakes can persist for long time periods which can be significant for the detection and tracking of underwater vehicles

- Wakes are significant for mixing and transport of heat and energy far downstream of objects like mountains and islands

Sample problem size, Pr=7 case from de Stadler et al., PF 22, 095102 2010:

1.88 billion grid points (DNS)
448 processors
14,000 CPU hours
Conclusions

Computational science has provided many opportunities and challenges for future fluid dynamics research

- Feature isolation and tracking has provided means of gaining both quantitative and qualitative information on coherent structures in geophysical turbulence

- Wavelets provide a means of splitting a 3D vorticity field into a coherent well-correlated field and incoherent random field with the coherent field containing significantly fewer wavelet modes

- Development of databases which can aid in analyzing the massive amounts of data generated by simulations greatly reducing post-processing time and headaches

- Development and maintenance of robust, highly-scalable simulators
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