A Prototype Finite Difference Model

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A Prototype Model

We will introduce a finite difference model that will serve to demonstrate what a computational scientist needs to do to take advantage of Distributed Memory computers using MPI

The model we are using is a two dimensional solution to a model problem for Ocean Circulation

The Stommel Problem

Wind-driven circulation in a homogeneous rectangular ocean under the influence of surface winds, linearized bottom friction, flat bottom and Coriolis force.

Solution: intense crowding of streamlines towards the western boundary caused by the variation of the Coriolis parameter with latitude

Governing Equations and Model Constants

$$\psi = 0$$

$$\gamma\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \beta\frac{\partial\psi}{\partial x} = f$$

$$f = -\alpha \sin\left(\frac{\pi y}{2L_y}\right)$$

$$\psi = 0 \qquad \psi = 0$$

$$L_x = L_y = 2000Km$$

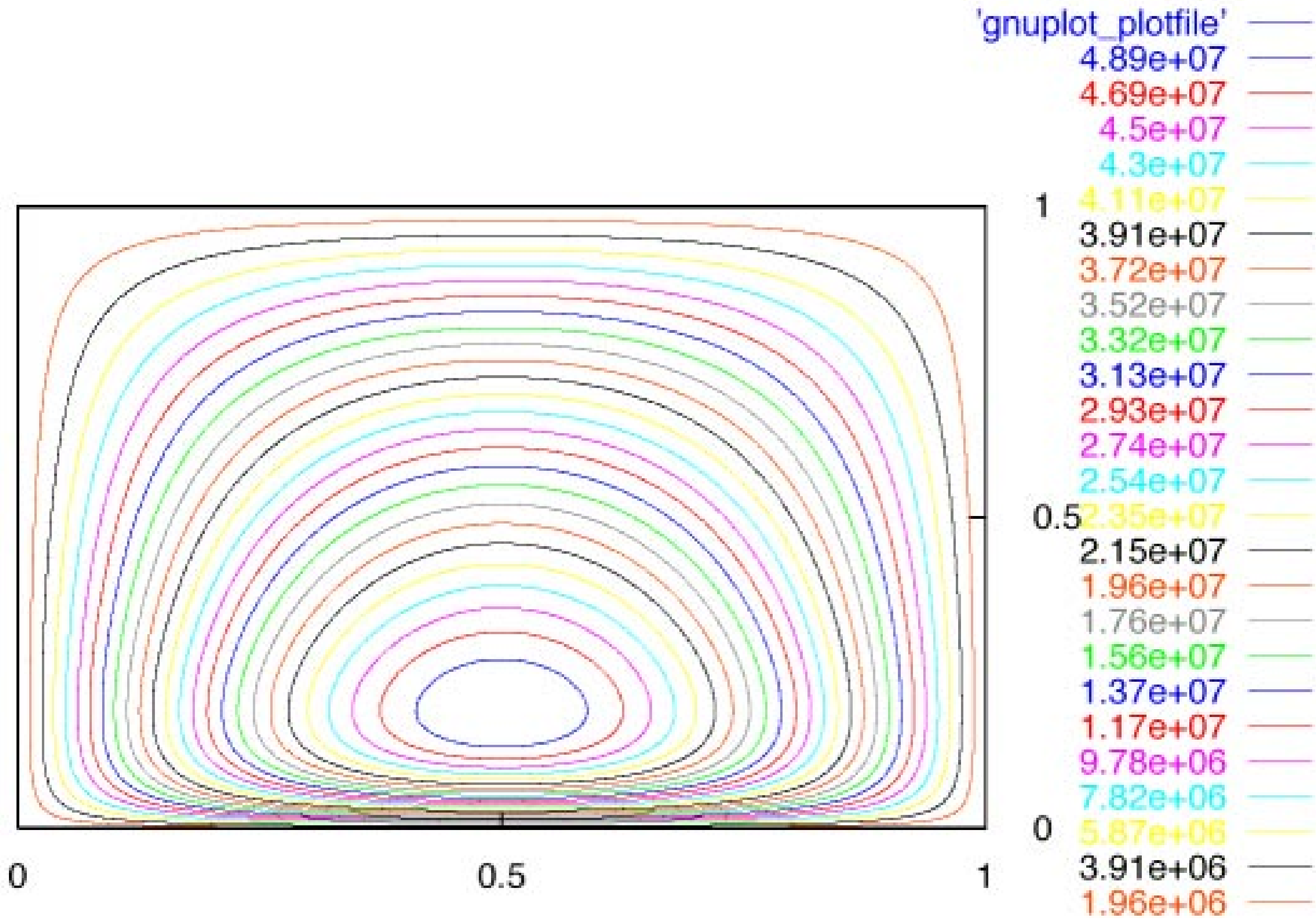
$$\gamma = 3 * 10^{(-6)}$$

$$\beta = 2.25 * 10^{(-11)}$$

$$\alpha = 10^{(-9)}$$

$$\psi = 0$$

Steady State Solution



Domain Discretization

Define a grid consisting of points (x_i, y_j) given by

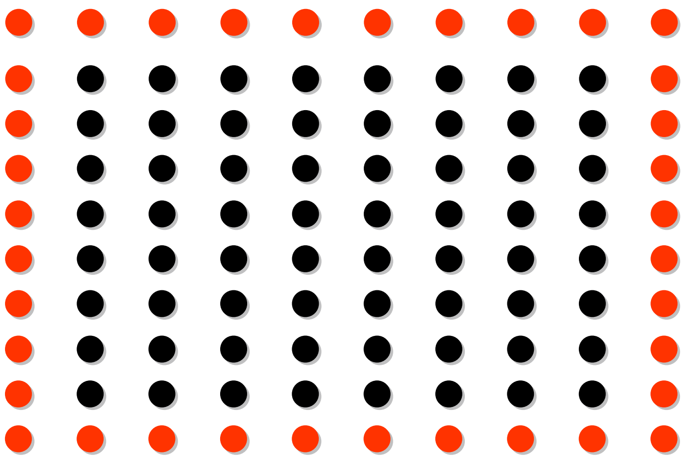
$$x_i = i\Delta x, i = 0, 1, \ldots\ldots, nx+1$$

$$y_j = j\Delta y, j = 0, 1, \ldots\ldots, ny+1$$

$$\Delta x = L_x / (nx + 1)$$

$$\Delta y = L_y / (ny + 1)$$

Domain Discretization



seek to find an approximation to $\psi(x_i, y_j)$ at points (x_i, y_j):

$$\psi_{i,j} \approx \psi(x_i, y_j)$$

Centered Finite Difference Scheme for the Derivative Operators

$$\frac{\partial \psi}{\partial x} \approx \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x}$$

$$\frac{\partial^2 \psi}{\partial x^2} \approx \frac{\psi_{i+1} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2 \psi}{\partial y^2} \approx \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2}$$

Governing Equation- Finite Difference Form

$$\psi_{i,j} = a_1\psi_{i+1,j} + a_2\psi_{i-1,j} + a_3\psi_{i,j+1} + a_4\psi_{i,j-1} - a_5 f_{i,j}$$

$$a_1 = \frac{\Delta y^2}{2(\Delta x^2+\Delta y^2)} + \frac{\beta\Delta x^2\Delta y^2}{4\gamma\Delta x(\Delta x^2+\Delta y^2)}$$

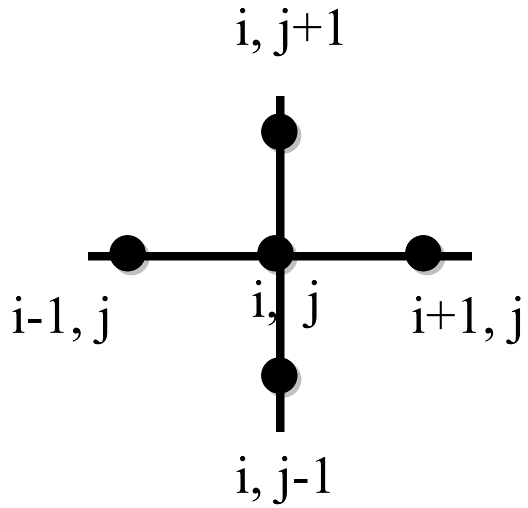
$$a_2 = \frac{\Delta y^2}{2(\Delta x^2+\Delta y^2)} - \frac{\beta\Delta x^2\Delta y^y}{4\gamma\Delta x(\Delta x^2+\Delta y^2)}$$

$$a_3 = \frac{\Delta x^2}{2(\Delta x^2+\Delta y^2)}$$

$$a_4 = \frac{\Delta x^2}{2(\Delta x^2+\Delta y^2)}$$

$$a_5 = \frac{\Delta x^2\Delta y^2}{2\gamma(\Delta x^2+\Delta y^2)}$$

Five-point Stencil Approximation for the Discrete Stommel Model



interior grid points:
i=1,nx; j=1,ny

boundary points:

(i,0) & (i,ny+1) ; i=1,nx

(1,0) & (nx+1,j) ; j=1,ny

$$\psi_{i,j} = a_1\psi_{i+1,j} + a_2\psi_{i-1,j} + a_3\psi_{i,j+1} + a_4\psi_{i,j-1} - a_5 f_{i,j}$$

$$\psi_{i,0} = \psi_{i,ny+1} = 0; \quad \psi_{0,j} = \psi_{nx+1,j} = 0;$$

Jacobi Iteration

Start with an initial guess for $(\psi_{i,j})$

do i = 1, nx; j = 1, ny

$$(\psi_{i,j})\_new \quad = \quad a_1(\psi_{i+1,j}) + a_2(\psi_{i-1,j}) + a_3(\psi_{i,j+1}) + a_4(\psi_{i,j-1}) - a_5 f_{i,j}$$

end do

Copy $(\psi_{i,j}) \quad = \quad (\psi_{i,j})\_new$

Repeat the process