MULTIPLE REACTIVITY CALCULATION USING SINGLE CORRELATED SAMPLING MONTE CARLO SIMULATION

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ABSTRACT Correlated sampling technique has been applied to the fission matrix approach for Monte Carlo eigenvalue calculation of reactivities (ΔK) due to small cross-section perturbations. The reference Monte Carlo simulation is performed in a system different from both the unperturbed and perturbed systems. This allows the determination of multiple ΔKs due to multiple cross-section perturbations with a single Monte Carlo simulation and a corresponding significant reduction in computational effort. The method has been tested for one energy group transport problems with isotropic scattering.

I. INTRODUCTION

For Monte Carlo reactivity calculation due to small cross-section perturbations, a correlated simulation between the unperturbed and the perturbed histories is necessary. Differencing the eigenvalues of two independent (unperturbed and perturbed) Monte Carlo simulations poses problems for a calculation of a small reactivity perturbation. This is due to the fact that the uncertainty associated with each of the two independent Monte Carlo eigenvalue simulations tends to be larger than the difference of the eigenvalues[1]. Also the conventional Monte Carlo source iteration technique to calculate the eigenvalue does not apply for eigenvalue perturbations except for a very limited class of problems[2]. The source iteration method follows an artificially stabilized neutron population from one generation to another; and it is not possible to calculate an eigenvalue perturbation by transferring correlated perturbed weights from one generation to the next. However, the Green's function or fission matrix approach is suitable for applying correlated sampling methods to calculate small perturbations in reactivity. In this approach, the fission source region is subdivided into volume elements and the mutual fission probabilities for the mesh elements are calculated using Monte Carlo simulation. The dominant eigenvalue of the resulting fission matrix constitutes the multiplication factor of the system. In this method only the fission matrix is evaluated using Monte Carlo simulation whereas the eigenvalue problem is solved using a conventional matrix iterative algorithm. Correlated sampling has been applied to the fission matrix approach where along with the unperturbed fission matrix another fission matrix, for the perturbed system, is generated from the correlated histories reacting with the perturbed system.

An efficient form of correlated sampling has been developed[3] where the Monte Carlo simulation is carried out in an artificial reference system which is different from both the unperturbed and perturbed systems. The fission matrices for the unperturbed and perturbed systems are formed by correlating those histories to the reference system's histories and by accounting for the differences in cross-sections. In this paper we report an extension of this idea for multiple reactivity calculations due to multiple cross-section perturbations, using a single Monte Carlo simulation. The Monte Carlo simulation is done in the artificial reference system and the unperturbed and multiple perturbed fission matrices are generated by correlating them to the reference system's histories. We have verified the accuracy of this method against a deterministic transport theory code for two dimensional Cartesian geometry perturbation problems in one energy group and with isotropic scattering. Since multiple reactivities, due to multiple perturbations, are calculated from single simulation a significant reduction in computational effort is observed. This is shown by comparing the computational time required for multiple reactivity calculation to that for single reactivity calculation.

II. MONTE CARLO EIGENVALUE CALCULATION FOR REACTIVITY PERTURBATION

There are two methods to calculate the multiplication constant K of a system using Monte Carlo methods[4]. One is the source iteration technique where K for a generation is obtained from a ratio of the cumulative total number of fission neutrons to the cumulative total number of source neutrons. The first generation source particles are started from an assumed spatial distribution. For subsequent generations, the source distribution is based on the fission neutron production distribution obtained from the previous generation. After each generation, some type of source normalization is performed to prevent population explosion or extinction. Some initial generations are discarded to eliminate the effect of the assumed first generation source distribution. The average K for the remaining generations gives an estimate of the multiplication constant along with the statistical uncertainties. This procedure is not suitable for eigenvalue perturbation calculation because perturbed weights, propagated from one generation to next, tend to grow and eventually all the pertinent information about the perturbation is lost.

Another procedure to calculate eigenvalue, using the Monte Carlo method is to make use of the Green's function or fission matrix approach. The Monte Carlo fission matrix method defines the multiplication constant as the eigenvalue of the homogeneous transport equation,

$$KQ(\vec{r}) = \int d\vec{r}\,' Q(\vec{r}\,') P(\vec{r}\,' \rightarrow \vec{r}) \quad , \tag{1}$$

where $Q(\vec{r})d\vec{r}$ is the number of fission neutrons produced in the differential volume $d\vec{r}$ about the position $\vec{r}$, $P(\vec{r}\,' \rightarrow \vec{r})d\vec{r}$ is the number of next generation fission neutrons produced in $d\vec{r}$ about $\vec{r}$ due to one fission neutron starting at $\vec{r}\,'$, and K is the multiplication factor, or the dominant eigenvalue of equation (1). A spatial grid (V_m; m=1,2,...,N) is imposed on all regions that contain a fission source. Then, integrating over a volume V_m, equation (1) yields:

$$KQ_m = \sum_{l=1}^{N} p_{l\,m} Q_l \quad , \tag{2}$$

where Q_m is the total number of fission neutrons produced in region m and $p_{l\,m}$ (elements of matrix P) is the expected number of fission neutrons produced in region m due to one fission neutron starting in region l. The matrix P is estimated during random walk within each fission generation. Once the $p_{l\,m}$'s have been evaluated using Monte Carlo method the eigenvalue problem of equation (2) can be solved using standard matrix iterative methods for the dominant eigenvalue. The calculation of the fission matrix constitutes an initial value problem and correlated sampling can be applied directly to it.

III. APPLICATION OF CORRELATED SAMPLING FOR REACTIVITY CALCULATION

In the correlated sampling technique for particle transport problems, the perturbed histories are forced to follow the same transition points in phase space as the unperturbed histories[5]. Appropriate weight factors are then used to correct the weight of the perturbed particles due to the perturbation. Regular correlated sampling techniques require that the Monte Carlo simulation is done in the unperturbed (up) system. It is assumed that another particle is being simulated simultaneously in the perturbed (p) system. Both the unperturbed and the perturbed particles start with a weight of unity. Whenever the unperturbed particle enters a perturbed region the weight of the perturbed particle is multiplied by a biasing factor,

$$B = \frac{F_p}{F_{up}} = \frac{{\Sigma_t}^p exp(-{\Sigma_t}^p l)}{{\Sigma_t}^{up} exp(-{\Sigma_t}^{up} l)} \quad , \tag{3}$$

where l is the distance traveled within the perturbed region and Σ_t is the macroscopic total cross-section. It is necessary that B and hence F_p and F_{up} remain finite i.e. $F_p > 0$ and $F_{up} > 0$. In some practical cases (e.g. void region) these conditions may not be satisfied. To avoid this situation a special form of correlated sampling is used where the Monte Carlo particle transport is simulated in a reference (ref) system which is generally different from both the unperturbed and perturbed systems. An added advantage of this method is that a proper choice of the reference system can avoid large fluctuations in biasing factors $\frac{F_{up}}{F_{ref}}$ and $\frac{F_p}{F_{ref}}$, and therefore reduce the uncertainty

involved with the differential effect. One way (not the only way)[6] to choose the reference system is to take the average cross-section of perturbed and unperturbed systems i.e.,

$$\Sigma_x^{ref} = \frac{1}{2}(\Sigma_x^p + \Sigma_x^{up}) \quad , \tag{4}$$

where x denotes different types of cross-sections. The Monte Carlo particle transport simulation is done only in the reference system with total macroscopic cross-section Σ_t^{ref}. The spatial collision distance s is sampled from ${\Sigma_t}^{ref} exp(-\Sigma_t^{ref} s)$. The biasing factor with which the weight of the unperturbed particle is multiplied is given by,

$$B_{up} = \frac{{\Sigma_t}^{up}}{{\Sigma_t}^{ref}} exp({\Sigma_t}^{ref} - {\Sigma_t}^{up})s \quad . \tag{5}$$

Similarly the biasing factor for the perturbed particle is,

$$B_p = \frac{{\Sigma_t}^{p}}{{\Sigma_t}^{ref}} exp({\Sigma_t}^{ref} - {\Sigma_t}^{p})s \quad . \tag{6}$$

Now assuming $\Sigma_t^p > \Sigma_t^{up}$, the biasing factor for the unperturbed history has a positive exponent and may assume large values if s is large. And similarly the biasing factor for the perturbed history may assume a very small value. To avoid these large fluctuations, a δ-scatter in the forward direction is added to all three total reaction cross-sections. For each of the three cases the δ-scatter is chosen in such a way that the sum of the total reaction cross-section and δ-scatter are the same for all three cases. This correlated sampling technique can be used to simultaneously estimate the three fission matrices for the reference, unperturbed and perturbed systems. The dominant eigenvalues of these three fission matrices can then be evaluated using conventional matrix iterative methods. Since the fission matrices are correlated it is possible to calculate $\Delta K = K^p - K^{up}$ for small cross-section perturbations. This efficient method has been used successfully to calculate reactivity[7] changes due to cross-section perturbation.

IV. MULTIPLE REACTIVITY CALCULATION

We have extended the correlated sampling fission matrix method to evaluate multiple reactivities (ΔK_i, i = 1,2,3,...,N) due to multiple small perturbations of the same unperturbed system. Multiple reactivities are calculated from a single Monte Carlo simulation. We apply correlated sampling to the fission matrix approach of eigenvalue calculation. To describe this method we assume that there is an unperturbed system and N small cross-section perturbations of this system lead to N perturbed systems denoted by p_i, i = 1,2,3,...,N.

A. Reference System

The reference system is chosen such that,

$$\Sigma_x^{ref} = \Sigma_x^{up} + \frac{1}{N}\sum_{i=1}^{N} \Delta\Sigma_x^{p_i} = \frac{1}{N}\sum_{i=1}^{N} \Sigma_x^{p_i} \quad , \tag{7}$$

where $\Delta\Sigma_x^{p_i} = \Sigma_x^{p_i} - \Sigma_x^{up}$ and x denotes a different cross-section type. It is possible to choose a different reference system and the optimum choice for reference system may vary from one problem to another. For all the numerical results shown in this paper we have used equation (7) to determine the reference system. The Monte Carlo particle tracking is done in the reference system. The spatial collision distance s is sampled from ${\Sigma_t}^{ref} exp(-\Sigma_t^{ref} s)$. The biasing factors for the unperturbed and the perturbed systems are given respectively by,

$$B_{up} = \frac{{\Sigma_t}^{up}}{{\Sigma_t}^{ref}} exp({\Sigma_t}^{ref} - {\Sigma_t}^{up})s \quad , \tag{8}$$

and

$$B_{p_i} = \frac{{\Sigma_t}^{p_i}}{{\Sigma_t}^{ref}} exp({\Sigma_t}^{ref} - {\Sigma_t}^{p_i})s \quad . \tag{9}$$

B. Forward δ-scatter

To avoid large difference between B_{up} and B_{pi}, a δ-scatter in the forward direction is added to the total reaction cross-sections for the reference, unperturbed and perturbed systems. The δ-scatter for the reference system (δ^{ref}) is chosen, depending upon the problem, such that,

$$\delta^{ref} > \mid \Sigma_t^{ref} - max(\Sigma_t^{p_i}) \mid \quad ; if\ max(\Sigma_t^{p_i}) > \Sigma_t^{up} \quad , \tag{10}$$

$$\delta^{ref} > \mid \Sigma_t^{ref} - \Sigma_t^{up} \mid \quad ; if\ \Sigma_t^{up} > max(\Sigma_t^{p_i}) \quad . \tag{11}$$

Then δ-scatter for the unperturbed system (δ^{up}) and perturbed systems (δ^{p_i}, i = 1,2,3,...,N) are chosen as,

$$\delta^{up} = \Sigma_t^{ref} + \delta^{ref} - \Sigma_t^{up} \quad , \tag{12}$$

$$\delta^{p_i} = \Sigma_t^{ref} + \delta^{ref} - \Sigma_t^{p_i} \quad . \tag{13}$$

The conditions imposed by equation (10) and (11) ensure that all the δ-scatters are positive quantities. In our test problems the above four equations were used to determine forward δ-scatter for reference, unperturbed and perturbed systems. The spatial collision distance s in the reference system is sampled from $({\Sigma_t}^{ref} + \delta^{ref}) exp(-(\Sigma_t^{ref} + \delta^{ref})s)$. Now the modified biasing factors for the unperturbed and the perturbed systems become,

$$B_{up} = \frac{({\Sigma_t}^{up} + \delta^{up}) exp(-(\Sigma_t^{up} + \delta^{up})s)}{({\Sigma_t}^{ref} + \delta^{ref}) exp(-(\Sigma_t^{ref} + \delta^{ref})s)} = 1 \quad , \tag{14}$$

$$B_{p_i} = \frac{({\Sigma_t}^{p_i} + \delta^{p_i}) exp(-(\Sigma_t^{p_i} + \delta^{p_i})s)}{({\Sigma_t}^{ref} + \delta^{ref}) exp(-(\Sigma_t^{ref} + \delta^{ref})s)} = 1 \quad . \tag{15}$$

Thus the large fluctuations in B_{up} and B_{p_i} are avoided.

C. ΔK Calculation

For the fission matrix eigenvalue calculation in the reference system, the first generation is started with an assumed source distribution and $W_{ref} = 1$, where W_{ref} is the weight of a reference system particle. Along with that, the initial weights of the unperturbed (W_{up}) and all the perturbed (W_{p_i}) particles are also set to one. Even though particle tracking simulation is only carried out in the reference system, the weights of the unperturbed and all the perturbed systems are modified by multiplying them with appropriate biasing factors B_{up} and B_{p_i} during simulation. At the end of the first generation, source normalization is done for the reference system particles to stabilize neutron population. The source normalization has no direct effect on other systems, except that the next generation unperturbed and perturbed systems' particles start out with the same weight as that of the normalized reference system particles. For the second generation the fission neutron production distribution obtained from the first generation is used for the reference system particles. The unperturbed and the perturbed systems' particles also use the same source distribution as that of the reference system for the second generation. This process is continued for specified number of generations. A few of the initial generations are discarded to avoid the bias due to the initial source guess.

Now the fission rate is determined, similar to equation (2), as the probability that a particle starting in volume element l generates $p_{l,m}$ particles in element m. For the reference system the matrix elements $p_{l,m}^{ref}$ of the reference fission matrix P^{ref} are scored as,

$$p_{l,m}^{ref} = p_{l,m}^{ref} + W^{ref} \frac{[\nu^{ref} \Sigma_f^{ref}]}{[\Sigma_t^{ref} + \delta^{ref}]} \quad , \tag{16}$$

where ν^{ref} is the number of particles emerging from a fission process in the reference system and Σ_f^{ref} is the macroscopic fission cross-section for the reference system. Simultaneously the fission matrix elements for the unperturbed and perturbed fission matrices, P^{up} and P^{p_i} respectively, are scored as,

$$p_{l,m}^{up} = p_{l,m}^{up} + W^{up} \frac{[\nu^{up} \Sigma_f^{up}]}{[\Sigma_t^{ref} + \delta^{ref}]} \quad , \tag{17}$$

$$p_{l,m}^{p_i} = p_{l,m}^{p_i} + W^{p_i} \frac{[\nu^{p_i} \Sigma_f^{p_i}]}{[\Sigma_t^{ref} + \delta^{ref}]} \quad . \tag{18}$$

The dominant eigenvalues K^{ref}, K^{up} and K^{p_i} of matrices P^{ref}, P^{up} and P^{p_i} respectively, are determined numerically. Then multiple reactivities due to multiple perturbations (p_i, i = 1,2,3,...,N) are calculated as,

$$\Delta K^i = K^{p_i} - K^{up} \quad . \tag{19}$$

The standard deviation of the single generation ΔK is given by,

$$\sigma = \sqrt{\frac{\sum_{n=1}^{M} \Delta K_n^2}{M-1} - \frac{(\sum_{n=1}^{M} \Delta K_n)^2}{M(M-1)}} \quad , \tag{20}$$

where M is the active number of fission generations. And the standard deviation of the mean is,

$$\sigma_m = \frac{\sigma}{M^{\frac{1}{2}}} \quad . \tag{21}$$

This σ_m is provided with the numerical Monte Carlo results in section V.

D. Variance Reduction

Survival biasing and Russian roulette techniques[8] are applied to the reference system particles. After determining the site of collision the survival chance of the particle in the reference system is sampled from,

$$p_s^{ref} = \frac{\Sigma_s^{ref} + \delta^{ref}}{\Sigma_t^{ref} + \delta^{ref}} \quad , \tag{22}$$

where Σ_s is the macroscopic scattering cross-section. This leads to the following biasing factors for the unperturbed and perturbed particles,

$$p_s^{up} = \frac{\Sigma_s^{up} + \delta^{up}}{\Sigma_t^{ref} + \delta^{ref}} \quad , \tag{23}$$

$$p_s^{p_i} = \frac{\Sigma_s^{p_i} + \delta^{p_i}}{\Sigma_t^{ref} + \delta^{ref}} \quad . \tag{24}$$

The weights of all the particles are reduced as follows at the site of the jth collision,

$$W_{ref}^j = W_{ref}^{j-1} p_s^{ref} \quad , \tag{25}$$

$$W_{up}^j = W_{up}^{j-1} p_s^{up} \quad , \tag{26}$$

$$W_{p_i}^j = W_{p_i}^{j-1} p_s^{p_i} \quad . \tag{27}$$

After the weight adjustment the Russian roulette method is applied to the reference system particles. In this procedure it is checked to determine if the reference system particle weight has fallen below some minimum value. A random number ξ (uniformly distributed between 0 and 1) is generated and compared to an input number θ which is typically between 2 and 10. If $\xi > \frac{1}{\theta}$, the reference system particle and along with that the unperturbed and all the perturbed particles are terminated. If $\xi \leq \frac{1}{\theta}$, the reference system particle and along with that the unperturbed and all the perturbed particles are continued with their weight multiplied by θ.

V. NUMERICAL RESULTS

We have implemented the correlated sampling method for multiple reactivity calculations in x-y geometry and one energy group with isotropic scattering. Multiple ΔKs due to multiple small cross-section perturbations were evaluated using this Monte Carlo test code and the results were compared to that of the TWODANT[9] code with

S_{16} Gauss-Legendre quadrature set and the convergence criterion for inner and outer iterations set at 10^{-12}. The dimensions of each square mesh for the TWODANT simulation are .125 mean free path X .125 mean free path. The homogeneous test problem is a square region with dimensions of 16.659cm X 16.659cm and cross-section perturbations were performed over the whole square region. The heterogeneous problem, with the same dimensions as the homogeneous one, has a center square region of 8.329cm X 8.329cm. For the heterogeneous problem, cross-section perturbations were performed only in the inner square region. All test cases have vacuum boundary conditions on four sides. Tables I (homogeneous problem, two ΔK calculation), II (heterogeneous problem, two ΔK calculation), III (homogeneous problem, three ΔK calculation) and IV (heterogeneous problem, three ΔK calculation) show comparisons of ΔK results between the Monte Carlo test code and the TWODANT code for different test problems. Errors ($|\frac{TWODANT-MonteCarlo}{TWODANT}|$) in ΔK are also given. In table V we show computational timing results for multiple ΔK calculation compared to a single calculation. These timing results are the average of several computer runs for the homogeneous test problems.

VI. DISCUSSION

Comparing relative errors of results between the TWODANT code and the Monte Carlo test code we observe that the results are in agreement with a maximum error of $\sim$4%. It appears that it is possible to calculate multiple reactivities due to multiple small perturbations using one Monte Carlo simulation for one energy group transport problems with isotropic scattering. We observe from the timing results in table V that, if applicable, this approach could result in significant saving in computational effort. The only extra computational effort involved for multiple ΔK evaluation compared to one ΔK evaluation, is in the number of times the matrix iterative algorithm is used. Computational time spent in this matrix iterative algorithm is insignificant compared to that spent in Monte Carlo particle tracking. This particular correlated sampling technique is applicable to problems where the perturbations are performed over large spatial regions. It should be noted that this methodology worked for relatively large perturbations (e.g. 10% or more change in ΔK) as well as small perturbations. One possible area of application would be to calculate reactivities due to variations in soluble boron concentrations.

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Table I. Two ΔK calculation for homogeneous case.

Unperturbed Σ_s=0.9,	cross-sections Σ_a=0.1,	Σ_t=1.0, $\nu\Sigma_f$=0.11	
Perturbed cross-sections	TWODANT ΔK	Two Correlated Monte Carlo ΔK	error (%)
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.108	-.016683	$-.016682 \pm .10E-4$	.006
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.1109	.0075073	$.0075070 \pm .48E-5$	.004
Σ_t=1.001, Σ_s=0.9 Σ_a=.101, $\nu\Sigma_f$=.11	-.0074674	$-.0076096 \pm .89E-5$	1.9
Σ_t=1.0001, Σ_s=0.9 Σ_a=.1001, $\nu\Sigma_f$=.11	-.0007522	$-.0007666 \pm .90E-6$	1.9
Σ_t=1.0, Σ_s=.901 Σ_a=.099, $\nu\Sigma_f$=.11	.007718	$.00773 \pm .10E-4$	.2
Σ_t=1.0, Σ_s=.899 Σ_a=.101, $\nu\Sigma_f$=.11	-.007590	$-.007609 \pm .10E-4$	.2
Σ_t=1.002, Σ_s=.86 Σ_a=.142, $\nu\Sigma_f$=0.149	-.003040	$-.002989 \pm .18E-3$	1.9
Σ_t=1.0001, Σ_s=.897 Σ_a=.1031, $\nu\Sigma_f$=0.114	.009408	$.009418 \pm .12E-4$	.1

Table II. Two ΔK calculation for heterogeneous case.

Unperturbed cross-sections Σ_s=0.9, Σ_a=0.1, $\nu\Sigma_f$=0.11			
Perturbed cross-sections	TWODANT ΔK	Two Correlated Monte Carlo ΔK	error (%)
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.108	-.010029	$-.00974 \pm .25E-4$	2.8
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.1109	.004567	$.004438 \pm .11E-4$	2.8
Σ_t=1.001, Σ_s=0.9 Σ_a=.101, $\nu\Sigma_f$=.11	-.004583	$-.004443 \pm .21E-4$	3.0
Σ_t=1.0001, Σ_s=0.9 Σ_a=.1001, $\nu\Sigma_f$=.11	-.0004619	$-.0004478 \pm .22E-5$	3.0
Σ_t=1.0, Σ_s=.901 Σ_a=.099, $\nu\Sigma_f$=.11	.004680	$.004560 \pm .13E-4$	2.5
Σ_t=1.0, Σ_s=.899 Σ_a=.101, $\nu\Sigma_f$=.11	-.004599	$-.004482 \pm .13E-4$	2.5
Σ_t=1.002, Σ_s=.86 Σ_a=.142, $\nu\Sigma_f$=0.149	.001951	$.001912 \pm .67E-4$	1.9
Σ_t=1.0001, Σ_s=.897 Σ_a=.1031, $\nu\Sigma_f$=0.114	.005779	$.005625 \pm .10E-4$	2.6

Table III. Three ΔK calculation for homogeneous case.

Unperturbed cross-sections Σ_t=1.0, Σ_s=0.9, Σ_a=0.1, $\nu\Sigma_f$=0.11			
Perturbed cross-sections	TWODANT ΔK	Three Correlated Monte Carlo ΔK	error (%)
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.108	-.016683	$-.016679 \pm .18E-4$	.02
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.1109	.0075073	$.0075058 \pm .83E-5$	.02
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.121	.091756	$.091738 \pm .10E-3$	.03
Σ_t=1.01, Σ_s=0.9 Σ_a=.11, $\nu\Sigma_f$=.11	-.06958	$-.07057 \pm .12E-3$	1.4
Σ_t=1.001, Σ_s=0.9 Σ_a=.101, $\nu\Sigma_f$=.11	-.007467	$-.007581 \pm .13E-4$	1.5
Σ_t=1.0001, Σ_s=0.9 Σ_a=.1001, $\nu\Sigma_f$=.11	-.0007522	$-.0007638 \pm .13E-5$	1.5
Σ_t=1.0, Σ_s=.9009 Σ_a=.0991, $\nu\Sigma_f$=.11	.006940	$.006944 \pm .10E-4$	.05
Σ_t=1.0, Σ_s=.901 Σ_a=.099, $\nu\Sigma_f$=.11	.007718	$.007723 \pm .11E-4$	.06
Σ_t=1.0, Σ_s=.899 Σ_a=.101, $\nu\Sigma_f$=.11	-.007590	$-.007595 \pm .11E-4$	.06
Σ_t=1.0003, Σ_s=.907 Σ_a=.0933, $\nu\Sigma_f$=.10699	.027766	$.027677 \pm .24E-4$	0.3
Σ_t=1.002, Σ_s=.86 Σ_a=.142, $\nu\Sigma_f$=0.149	.003040	$.003001 \pm .75E-4$	1.2
Σ_t=1.0001, Σ_s=.897 Σ_a=.1031, $\nu\Sigma_f$=0.114	.0094077	$.0094058 \pm .52E-5$	.02

Table IV. Three ΔK calculation for heterogeneous case.

Unperturbed Σ_s=0.9,	cross-sections Σ_a=0.1,	Σ_t=1.0, $\nu\Sigma_f$=0.11	
Perturbed cross-sections	TWODANT ΔK	Three Correlated Monte Carlo ΔK	error (%)
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.108	-.010029	$-.009713 \pm .36E-4$	3.1
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.1109	.004567	$.004422 \pm .16E-4$	3.1
Σ_t=1.0, Σ_s=0.9 Σ_a=0.1, $\nu\Sigma_f$=.121	.05789	$.05606 \pm .19E-3$	3.1
Σ_t=1.01, Σ_s=0.9 Σ_a=.11, $\nu\Sigma_f$=.11	-.04248	$-.04128 \pm .12E-3$	2.8
Σ_t=1.001, Σ_s=0.9 Σ_a=.101, $\nu\Sigma_f$=.11	-.004583	$-.004455 \pm .13E-4$	2.8
Σ_t=1.0001, Σ_s=0.9 Σ_a=.1001, $\nu\Sigma_f$=.11	-.0004619	$-.0004490 \pm .12E-5$	2.7
Σ_t=1.0, Σ_s=.9009 Σ_a=.0991, $\nu\Sigma_f$=.11	.004208	$.004133 \pm .13E-4$	1.8
Σ_t=1.0, Σ_s=.901 Σ_a=.099, $\nu\Sigma_f$=.11	.004680	$.004596 \pm .14E-4$	1.8
Σ_t=1.0, Σ_s=.899 Σ_a=.101, $\nu\Sigma_f$=.11	-.004599	$-.004517 \pm .14E-4$	1.7
Σ_t=1.003, Σ_s=.907 Σ_a=.096, $\nu\Sigma_f$=.10699	.0034704	$.0033154 \pm .56E-5$	4.4
Σ_t=1.002, Σ_s=.86 Σ_a=.142, $\nu\Sigma_f$=0.149	.001951	$.001992 \pm .43E-4$	2.1
Σ_t=1.0001, Σ_s=.897 Σ_a=.1031, $\nu\Sigma_f$=0.114	.0057797	$.0056409 \pm .73E-5$	2.4

Table V. Timing results for ΔK calculations.

# of ΔK calculations	Time (sec) normalized to one ΔK calculation
1	1.0
2	1.14
3	1.25