CHEM 190 and CHEM 290

K. Lindenberg and K. Baldridge Spring 2001

Homework No. 7
Due May 24 in class

1. **Textbook Chapter 6 No. 3**
2. **Textbook Chapter 6 No. 6**
3. **Textbook Chapter 6 No. 22**
4. **Textbook Chapter 6 No. 39**
5. **Textbook Chapter 6 No. 67**
6. **Variation of Textbook Example Chapter 6 No. 38**: On an exam with 100 true-false questions 3 members of a class of 30 scored at least 80% correct. Can this result be ascribed to chance or did these students demonstrate a knowledge of the material?
7. **Textbook Chapter 6 No. 77**
8. **Textbook Chapter 6 No. 92**
9. **Textbook Chapter 6 No. 95**
10. **Textbook Chapter 6 No. 105**

ONLY FOR CHEM 290 STUDENTS
Due May 31 in class

1. This problem deals with "discrete Fourier transforms" and the associated problem of "aliasing." In turn, this is related to the "Fast Fourier Transform," a computational method that some of you may be using already in your research.

   Reconstruct the function $f(t) = \sin\, t$ in the interval $0 < t < \pi$ via a discrete Fourier transform on four points: 0,1,2, and 3. Compare the reconstructed function to the original function.

2. In a resonant cavity an electromagnetic oscillation of frequency ω_0 dies out as

$$A(t) = A_0 e^{-\omega_0 t/2Q} e^{-i\omega_0 t}, \quad t > 0.$$

   (Take $A(t) = 0$ for $t < 0$.) The parameter Q is a measure of the ratio of stored energy to energy loss per cycle. Show that the frequency distribution of the oscillation, $a^*(\omega)a(\omega)$, is

$$a^*(\omega)a(\omega) = \frac{A_0^2}{2\pi} \frac{1}{(\omega - \omega_0)^2 + (\omega_0/2Q)^2},$$

   where $a(\omega)$ is the Fourier transfrom of $A(t)$. *Note*: The larger Q is, the sharper is the resonance line.

3.

(a) A rectangular pulse is described by

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a. \end{cases}$$

Show that the Fourier exponential transform is

$$F(t) = \sqrt{\frac{2}{\pi}} \frac{\sin at}{t}.$$

Here is the single slit diffraction problem of physical optics. The slit is described by $f(x)$. The diffraction pattern *amplitude* is given by the Fourier transform $F(t)$.

(b) Use the Parseval relation to show that

$$\int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2} dt = \pi.$$